Ensuring DNN Solution Feasibility for Optimization Problems with Linear Constraints

TIANYU ZHAO, City University of Hong Kong
XIANG PAN, The Chinese Unversity of Hong Kong
MINGHUA CHEN, City University of Hong Kong
STEVEN H. LOW, California Institute of Technology

1 EXTENDED ABSTRACT

Recently, there have been increasing interests in employing neural networks, including deep neural networks (DNN), to solve constrained optimization problems in various problem domains, especially those needed to be solved repeatedly in real-time. The idea behind these machine learning approaches is to leverage the universal approximation capability of DNNs [5] to learn the mapping between the input parameters to the solution of an optimization problem. Then one can directly pass the input parameters through the trained DNN to obtain a quality solution much faster than iterative solvers. Researchers have developed DNN schemes to solve essential optimal power flow problems in grid operation with sub-percentage optimality loss and several orders of magnitude speedup as compared to conventional solvers [1, 3, 4, 8] and for real-time power control and beam-forming design [9] problems in wireless communication in a fraction of time used by existing solvers.

Despite these promising results, however, a major criticism of DNN and machine learning schemes is that they usually cannot guarantee the solution feasibility with respect to all the inequality and equality constraints of the optimization problem [8, 11]. This is due to the inherent neural network prediction errors. Existing works address the feasibility concern mainly by incorporating the constraints violation (e.g., a Lagrangian relaxation to compute constraint violation with Lagrangian multipliers) into the loss function to guide the DNN training. These endeavors, while generating great insights to the DNN design and working to some extent in case studies, can not guarantee the solution feasibility without resorting to expensive post-processing procedures, e.g., feeding the DNN solution as a warm start point into an iterative solver to obtain a feasible solution. To date, it remains a largely open issue of ensuring DNN solution feasibility for constrained optimization problems.

In this paper, we address this challenge for general Optimization Problems with Linear (inequality) Constraints (OPLC). Since linear equality constraints can be exploited to reduce the number of decision variables without losing optimality (and removed), it suffices to focus on problems with inequality constraints. We make the following contributions.

2 CONTRIBUTIONS AND RESULTS

We propose *preventive learning* as the first framework to ensure DNN solution feasibility for OPLC without post-processing.[1] We systematically calibrate inequality constraints used in DNN training, thereby anticipating prediction errors and ensuring the resulting DNN solutions remain feasible.

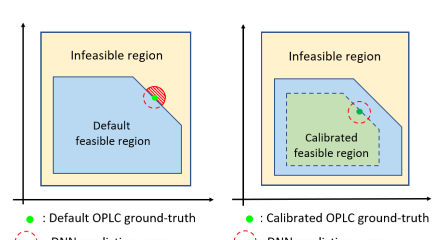

Fig. 1. Illustration of constraints calibration

▷ First, we determine the maximum calibration rate for inequality constraints, i.e., the rate of adjusting (reducing) constraints limits that represents the room for (prediction) errors without violating constraints, so that solutions from a preventively-trained DNN using the calibrated constraints respect the original constraints for all possible inputs. See Fig. 1 for illustrations.

[1]The results presented in this paper are reproduced from [10].

Table 1. Performance comparison with SOTA DNN schemes in light-load and heavy-load regimes.

Case	Scheme	Average speedups		Feasibility rate (%)		Optimality loss (%)		Worst-case violation (%)	
		light-load	heavy-load	light-load	heavy-load	light-load	heavy-load	light-load	heavy-load
Case30	DNN-P	×85	×86	100	88.12	0.02	0.03	0	5.43
	DNN-D	×85	×84	100	93.36	0.02	0.03	0	11.19
	DNN-W	×0.90	×0.86	100	100	0	0	0	0
	DNN-G	×24	×26	100	100	0.13	0.04	0	0
	DeepOPF+	×86	×93	100	100	0.03	0.09	0	0
Case118	DNN-P	×137	×125	68.84	54.92	0.17	0.21	19.5	44.8
	DNN-D	×138	×124	73.42	55.37	0.20	0.24	16.69	43.1
	DNN-W	×2.08	×2.26	100	100	0	0	0	0
	DNN-G	×26	×16	100	100	1.29	0.39	0	0
	DeepOPF+	×202	×228	100	100	0.37	0.41	0	0
Case300	DNN-P	×115	×98	91.29	78.42	0.06	0.08	261.1	443.0
	DNN-D	×115	×102	91.99	82.92	0.07	0.07	231.6	348.1
	DNN-W	×1.04	×1.08	100	100	0	0	0	0
	DNN-G	×2.44	×2.65	100	100	0.32	0.06	0	0
	DeepOPF+	×130	×138	100	100	0.10	0.06	0	0

▹ Second, we determine a sufficient DNN size so that with preventive learning there exists a DNN whose worst-case violation on calibrated constraints is smaller than the maximum calibration rate, thus ensuring DNN solution feasibility, i.e., DNN's output always satisfies the inequality constraints for any input. We then directly construct a provable feasibility-guaranteed DNN model.

▹ Third, observing the feasibility-guaranteed DNN may not achieve strong optimality result, we propose an adversarial training algorithm, called *Adversarial-Sample Aware* algorithm to further improve its optimality without sacrificing feasibility guarantee and derive its performance guarantee.

▹ We apply the framework to design a DNN scheme, DeepOPF+, to solve DC optimal power flow (DC-OPF) problems in grid operation. We compare DeepOPF+ with the conventional iterative OPF solver Pypower and four DNN based schemes DNN-P/DNN-D/DNN-W/DNN-G adapted from [7]/[3]/[2]/[6]. Simulation results over IEEE 30/118/300-bus test cases show that it outperforms existing strong DNN baselines in ensuring 100% feasibility and attaining consistent optimality loss (<0.19%) and speedup (up to ×228) in both light-load and heavy-load regimes.

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