Value-Oblivious Secretaries with Advice

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In the value-oblivious secretary problem, candidates arrive in a uniformly random order, and the decision maker only knows the relative ranks of candidates instead of their exact values. The objective is to maximize the probability of selecting the best candidate. We study this problem with *advice*, where a possibly erroneous prediction of the best candidate's position is provided. We design deterministic and randomized algorithms based on the classical wait-and-accept strategy, using a novel optimization-based framework that balances consistency and robustness. Our approach extends naturally to variants such as the multi-choice and rehiring secretary problems.

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1 INTRODUCTION

Learning-augmented algorithms aim to bridge worst-case and data-driven analysis by incorporating predictions, which are potentially derived from machine learning, into algorithm design. When accurate, such predictions can reveal structural information and significantly improve performance. However, a key challenge lies in designing algorithms that can benefit from accurate predictions (i.e., consistency) while remaining resilient to incorrect ones (i.e., robustness).

We focus on learning-augmented algorithms in the value-oblivious setting of the secretary problem [6] . In this setting, candidates arrive in a uniformly random order, and the goal is to select the best one when only knowing the arrivals' relative ranks. While recent work has primarily explored value-based secretary problems, where predictions about the numerical values of arriving elements guide decisions [1, 3–5, 7], the value-oblivious setting, where only relative rankings are observable and no explicit value information is available, remains underexplored in this context [2].

This work explores how potentially unreliable predictions about the position of the best candidate, rather than their value, can be used to improve performance in this setting. We introduce a new model, Value-Oblivious Secretaries with Advice (**VoSA**), and design algorithms that balance consistency and robustness. Our approach leverages the stochastic structure of the secretary problem and offers novel techniques applicable to several of its variants.

2 ALGORITHMIC APPROACH

We propose two algorithms for VoSA setting. Our goal is to balance *consistency* and *robustness*.

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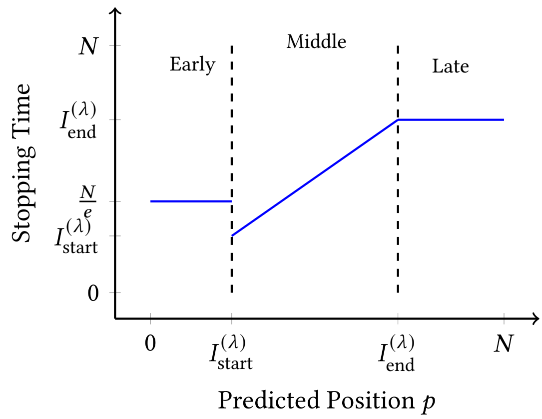

Fig. 1. Stopping time used by D-WnA as a function of the predicted position p.

2.1 Deterministic Algorithm

We begin by proposing a deterministic algorithm D-WnA, which adjusts its stopping strategy based on the predicted position p of the best candidate. The goal is to leverage correct predictions for improved performance (consistency), while maintaining guarantees on success when predictions are inaccurate (robustness). D-WnA partitions the prediction space into three regions, parameterized by a confidence parameter $\lambda \geq 1$. Specifically, it defines an interval $I = [I_{\text{start}}^{(\lambda)}, I_{\text{end}}^{(\lambda)}]$, where $I_{\text{start}}^{(\lambda)} = \lceil \exp\left(W_{-1}\left(\frac{-1}{\lambda e}\right)\right) \cdot N \rceil + 1$, and $I_{\text{end}}^{(\lambda)} = \lfloor \exp\left(W_0\left(\frac{-1}{\lambda e}\right)\right) \cdot N \rfloor + 1$, and W_{-1}, W_0 are the two real branches of the Lambert W function. We define WnA(x) as the algorithm that observes the first x candidates and then selects the next candidate better than all seen so far.

D-WnA. If $p < I_{\text{start}}^{(\lambda)}$, the algorithm ignores the prediction and uses the classical WnA$(\lfloor N/e \rfloor)$ strategy. If $p \in [I_{\text{start}}^{(\lambda)}, I_{\text{end}}^{(\lambda)}]$, it stops at time $p - 1$ and accepts the next best-so-far candidate. If $p > I_{\text{end}}^{(\lambda)}$, it caps the observation window at $I_{\text{end}}^{(\lambda)} - 1$. This piecewise stopping policy is illustrated in Figure 1. The algorithm behaves conservatively for unreliable predictions and fully exploits accurate ones within a controlled region.

Theorem 2.1. *For a given $\lambda \geq 1$, D-WnA is $(\frac{L^{(\lambda)}}{N} + \frac{1}{\lambda e})$-consistent and $\frac{1}{\lambda e}$-robust, where $L^{(\lambda)} = I_{end}^{(\lambda)} - I_{start}^{(\lambda)}$. In addition, D-WnA achieves the Pareto-optimal trade-off among all deterministic algorithms.*

2.2 Randomized Algorithm

To address the limitations of deterministic strategies—particularly their inability to select early-arriving candidates—we propose a randomized meta-algorithm, **R-WnA**, for the value-oblivious secretary problem with advice. This method introduces randomness in the selection rule to allow for *nonzero success probability across all arrival positions*, overcoming fairness issues and hard selection boundaries of deterministic approaches.

Core Idea. For analytical convenience, we switch to a continuous setting by normalizing candidate indices and prediction values to the interval $[0, 1]$, a transformation made without loss of generality, as results in the secretary problem are asymptotic. Given a prediction $p \in [0, 1]$ for the position of the best candidate, R-WnA selects a stopping point $t \in [0, 1]$ *randomly* according to a probability distribution $\psi_p(t)$. It then applies a wait-and-accept strategy. The randomized algorithm R-WnA is shown in Algorithm 1.

Algorithm 1 Randomized Wait-and-Accept (R-WnA)

1: **Input:** Prediction $p \in [0,1]$; family of probability density functions $\{\psi_p(t)\}$
2: Sample $t \sim \psi_p(t)$
3: Run $\text{WnA}(t)$: Observe and reject the first $\lfloor t \cdot N \rfloor$ candidates; then select the first candidate better than all seen so far

The family of distributions $\{\psi_p\}$ is designed to balance the consistency that maximizes the success probability when the prediction is accurate, and the robustness that ensures a competitive ratio of at least $1/(\lambda e)$ when predictions are inaccurate. In particular, for a given prediction p, the distribution is derived by solving the following optimization problem:

$$\max_{\psi_p(t)} \quad \int_0^p \psi_p(t) \cdot \frac{t}{p}\, dt \tag{1a}$$

$$\text{s.t.} \quad \int_0^1 \psi_p(t) \cdot t \ln \tfrac{1}{t}\, dt \geq \tfrac{1}{\lambda e}, \tag{1b}$$

$$\int_0^1 \psi_p(t)\, dt = 1, \quad \psi_p(t) \geq 0 \text{ on } [0,1]. \tag{1c}$$

The objective function represents the probability of selecting the best candidate when the prediction p is accurate. The first constraint enforces a lower bound on the expected *robustness*, measured by a known function $t \ln \frac{1}{t}$ that captures how reliably the algorithm performs under worst-case arrival scenarios. The remaining constraints ensure that $\psi_p(t)$ is a valid probability distribution.

Theorem 2.2. *For a given $\lambda \geq 1$, R-WnA is $\eta(\lambda)$-consistent and $\frac{1}{\lambda e}$-robust, where $\eta(\lambda)$ is given by*

$$\eta(\lambda) = \int_0^{I_{start}^{(\lambda)}} \frac{\frac{1}{e}\left(1 - \frac{1}{\lambda}\right)}{\frac{1}{e} - t \ln \frac{1}{t}}\, dt + (I_{end}^{(\lambda)} - I_{start}^{(\lambda)}) + \frac{1}{\lambda e}.$$

Theoretical analysis shows that R-WnA improves over deterministic baselines, particularly when handling early predictions. Our learning-augmented framework is flexible and can be extended to other variants of the secretary problem, such as the multi-choice and rehiring settings. In both cases, we adapt our optimization-based approach to preserve consistency-robustness trade-offs under additional structural constraints, demonstrating the general applicability of our method.

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